

4 Integralrechnung

Englische Aufgaben

4.1 Which of the functions F are antiderivatives of the given polynomial, trigonometric and exponential functions f , respectively?

a. $f(x) = 3x^2$

A $F(x) = 9x^2$

B $F(x) = 0.75x^4$

C $F(x) = 3x^4 - 7$

D $F(x) = \frac{3}{4x^4} + 12$

b. $f(x) = \sin(3x)$

A $F(x) = -\cos\left(\frac{3}{2}x^2\right)$

B $F(x) = \frac{-\cos(3x^2)}{3}$

C $F(x) = -\frac{1}{3}\cos(3x)$

D $F(x) = \cos(3x)$

c. $f(x) = e^{3x}$

A $F(x) = e^{\frac{3x^2}{2}}$

B $F(x) = 3e^{3x}$

C $F(x) = \frac{1}{3}e^{3x}$

D $F(x) = \frac{e^{3x+5}}{3}$

[antiderivative ... Stammfunktion]

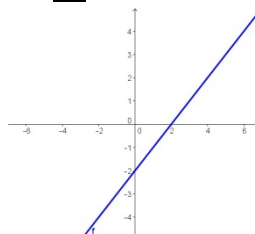
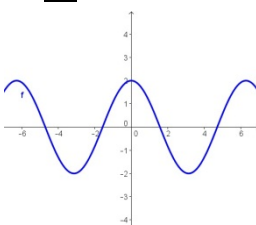
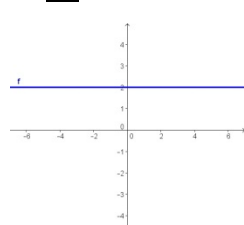
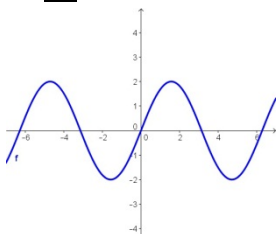
4.2 Match each function f to the graph of an appropriate primitive integral F .

a. ☐

b. ☐

c. ☐

d. ☐

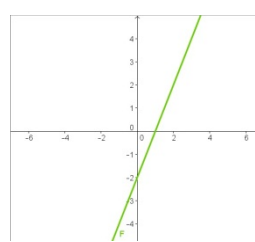
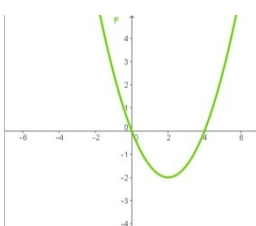
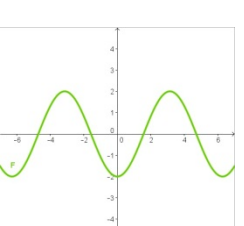
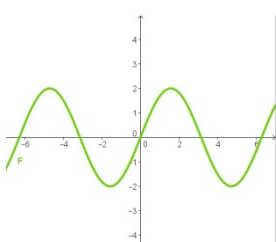


A

B

C

D



[primitive integral ... Stammfunktion]

4.3 Draw the graph of an antiderivative F of the function f satisfying the condition $F(a) = b$.

a. $f(x) = 3x^2$; $a = 0, b = -2$

b. $f(x) = \cos x$; $a = \frac{\pi}{2}, b = 5$

c. $f(x) = 8x + 8$; $a = -4, b = 0$

4.4 Compute the indefinite integral.

a. $\int 3x^2 - 5x + \frac{3}{x} dx$

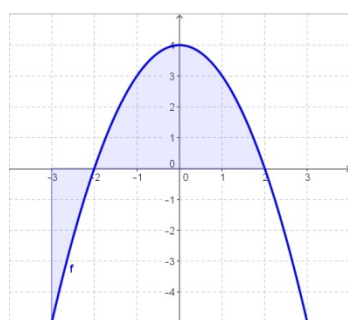
b. $\int 3xy^2 - 5yx^3 + y dy$

c. $\int 5ab^5 - a + \sin(ab) db$

[indefinite integral ... unbestimmtes Integral]

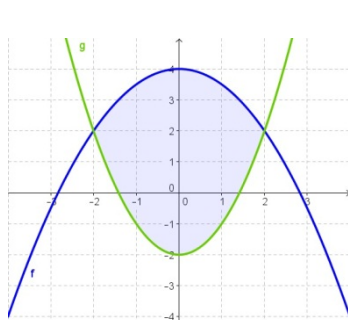
4.5 Write down a formula using definite integrals for computing the coloured area and compute the area.

a.



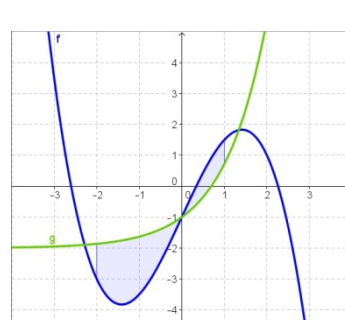
$f(x) = -x^2 + 4$

b.



$f(x) = -\frac{1}{2}x^2 + 4, g(x) = x^2 - 2$

c.



$f(x) = -\frac{1}{2}x^3 + 3x - 1, g(x) = e^x - 2$

[definite integral ... bestimmtes Integral; area ... Fläche]

4 Integralrechnung

Englische Aufgaben

- 4.6** Plot the function $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^4 - 5x^2 + 4$. Decide whether the following statements are true or false.

a. $\int_{-2}^1 f(x) dx = \int_1^2 f(x) dx$

b. The area enclosed by f and the x -axis is $A = \int_{-2}^2 f(x) dx$.

c. $\int_{-1}^1 f(x) dx = 2 * \int_0^1 f(x) dx$

d. $\int_0^2 f(x) dx < 0$

f. The area enclosed by f and the x -axis in $[-2; 0]$ is $\left| \int_{-2}^{-1} f(x) dx \right| = \left| \int_{-1}^0 f(x) dx \right|$.

[to enclose ... einschließen; x -axis ... x -Achse]

- 4.7** The acceleration of a car at t seconds is $a(t) = -0.0003t^2 + 0.08t + 2$ for t in $[0; 14]$.

a. Find a function v for t in $[0; 14]$ describing the cars' velocity at time t in m/s, if the cars' velocity at $t = 0$ is zero, and compute the velocity at $t = 7$.

b. Find a function s which for all t in $[0; 14]$ gives the covered distance in $[0; t]$ in meters.

c. Compute the distance the car has covered after 12 sec.

d. After 14 seconds the car slows down again. The deceleration due to braking is 6.5 m/s^2 . Find a function v_b which gives the velocity of the car at time t during the slowing-down process. Compute the time until the car stops as well as the covered distance.

[acceleration ... Beschleunigung; covered distance ... zurückgelegte Strecke; to slow down ... bremsen; deceleration due to braking ... Bremsverzögerung; slowing-down process ... Bremsvorgang]

- 4.8** Compute the mean value of $f(x) = 3x^3 - 15x$ in $[0; 4]$.

- 4.9** A cup of tea is left to cool down. The process of cooling down is described by the function T with $T(t) = 63e^{-0.15t} + 25$, where t is the time in minutes and $T(t)$ the temperature of the tea in $^{\circ}\text{C}$.

a. Compute the temperature of T at $t = 0$ and at $t = 10$.

b. How long does it take until the temperature has dropped to approximately 30°C ?

c. Compute the average temperature of the tea in the first 8 minutes.

[to leave to cool down ... abkühlen lassen; to drop ... absinken]

- 4.10** We know several function values of a continuous function f : $f(0) = 0$; $f(3) = 46.5$; $f(6) = 228$.

a. Compute the integral approximately using (1) the midpoint rule, (2) the trapezoidal rule and (3) Simpson's rule.

b. Assume, the equation of f is given by $f(x) = \frac{1}{2}x^3 + 3x^2 + 2x$. Compute the exact integral of f and the absolute error between each approximation and the true value of the integral.

[midpoint rule (or rectangle rule) ... Rechtecksregel; trapezoidal rule ... Trapezregel; Simpson's rule1 ... Kepler'sche Fassregel; remark ... Bemerkung]

- 4.11** Use the composite Simpson's rule and compute the integral $\int_0^6 x^3 e^{-x} dx$ for $n = 3$.

¹ **Remark:** In German, the formula $\int_a^b f(x) dx \approx \frac{1}{6} * (b - a) * \left[f(a) + 4 * f\left(\frac{a+b}{2}\right) + f(b) \right]$ is called *Kepler'sche Fassregel* and the formula $\int_a^b f(x) dx \approx \frac{b-a}{6n} [f(a) + 4 * f(a_1) + 2 * f(a_2) + \dots + 4 * f(a_{2n-1}) + f(b)]$ is called *zusammengesetzte Fassregel* or *Simpson-Regel*. However, in English the formulas are called *Simpson's rule* and *composite Simpson's rule*, respectively.