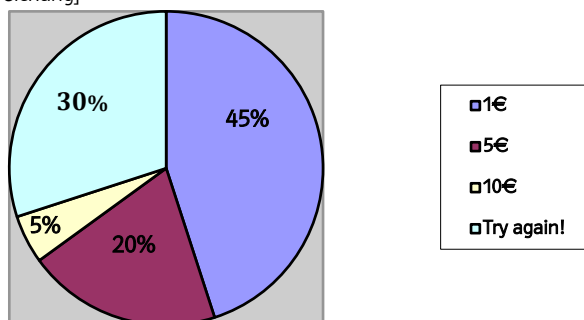


## 2 Zufallsvariable

### Englische Aufgaben

- 2.1** To win at a wheel of fortune is a game of luck. In the picture below the prizes and the probabilities of winning are shown. The wager for spinning the wheel once is 2€. If one reaches the field "Try again!", he is allowed to spin the wheel one more time, without additional payment. However, if the wheel stops at "Try again!" a second time, the game is lost. Let  $X$  be the discrete random variable denoting the winnings.
- Write down the set of events  $E_x$  for the winnings.
  - Assign each element of  $E_x$  the corresponding probability and draw the probability distribution.
  - Compute the expected value and the standard deviation of  $X$ , (1) without (2) with considering the wager.

[wheel of fortune ... Glücksrad; game of luck ... Glücksspiel; wager ... Einsatz; random/stochastic variable ... Zufallsvariable; set of events ... Ereignismenge; probability distribution ... Wahrscheinlichkeitsverteilung; expected value ... Erwartungswert; standard deviation ... Standardabweichung]



- 2.2** Several information about the winning possibilities can be found on the back of a lottery scratch card. The winning pyramid is shown in the table below. The current edition consists of 1.1 million scratch tickets, which are sold for 3€ per card.
- Compute the probability that a scratch card wins a prize.
  - Compute the probability to get a prize higher than the wager paid for the scratch card.
  - Compute the expected value for the prize of such a scratch card.

winning pyramid	
quantity/series	prize in EUR
3 x	50 000,-
10 x	5 000,-
100 x	500,-
5 000 x	25,-
12 000 x	15,-
30 000 x	9,-
90 000 x	6,-
205 000 x	3,-

[lottery scratch card/scratch ticket/scratch card ... Rubbellos; wager ... Einsatz]

- 2.3** Carl, George and Shawn spice up their training session with a game: they have four sheets of paper numbered from 1 to 4 in a bag. In turns, each boy draws two numbers. If one of the numbers is a 1, the boy doesn't have to do anything at all, otherwise, they are added up. If the total is an even number, the participant has to do the same amount of push ups, if the sum is odd, the participant has to do twice the number of push ups.
- Display the possible outcomes (i.e., the number of push ups a boy has to do) and their probabilities in a table.
  - Compute expected value, variance and standard deviation for the number of push ups a boy has to do.
  - It's Shawn's turn. Compute the probability that he has to do 10 push ups.

[even ... gerade; odd ... ungerade; push ups ... Liegestütz]

- 2.4** In sixth grade of a high school, 78% of the pupils go to school by bus every day. Compute the probability that among a random sample of 5 students...
- ... two students go to school by bus.
  - ... at least one goes to school by bus.
  - ... more than three go to school by bus.
  - Compute the expected value and the standard deviation for the number of pupils going to school by bus in this random sample.

[random sample ... Stichprobe; at least ... mindestens]

## 2 Zufallsvariable

### Englische Aufgaben

- 2.5** At a test consisting of twelve single choice questions, at least eight have to be answered correctly to pass the test. There are five answers to each question, of which exactly one is correct. Carla knows the correct answers to five of the questions. She ticks the other answers randomly. Compute the probability that ...
- ...none of the randomly checked questions is correctly answered.
  - ...Carla passes the test.
  - Compute the expected value and the standard deviation for correctly answered questions in Carlas situation.  
[to tick/to check ... ankreuzen]
- 2.6** Examine whether the given function  $f$  is a probability density function for the continuous random variable  $X$  in the domain  $D_x$ . If  $f$  is a probability density function, compute the corresponding distribution function  $F$ . Reason your decision.
- $f(x) = \frac{1}{2496} \cdot (3x^2 + 10x + 4)$ ,  $D_x = [0; 12]$
  - $f(x) = 1 - 0.25x$ ,  $D_x = [0; 4]$
  - $f(x) = \frac{2}{3} \cdot (2 - x)$ ,  $D_x = [0; 3]$
- [probability density function ... (Wahrscheinlichkeits-)Dichtefunktion; continuous random variable ... stetige Zufallsvariable; distribution function ... Verteilungsfunktion]
- 2.7** Find a number  $a$  such that  $f(x) = a \cdot (-x^2 + 64)$  is a probability density function for  $X \in [1; 8]$ .
- 2.8** Either a probability density function  $f$  or a distribution function  $F$  is known for  $X \in [0; 10]$ . Compute **(1)** the probabilities  $P(X \geq 7)$  and  $P(3 < X < 8)$ , **(2)** the expected value, **(3)** the median, and **(4)** the variance of  $X$ .
- $F(x) = 0.001 \cdot x^3$
  - $f(x) = \frac{x}{50}$
- 2.9** The lifespan of a new kind of refrigerators may be described by the probability density function  $f(x) = \frac{1}{10} e^{-\frac{x}{10}}$  and the corresponding random variable  $X$  indicates the durability in years.
- Compute the median as well as the upper and lower quartile.
  - Compute the expected value and compare it to the median.
  - Calculate the point of time from which only 5% of the refrigerators are functioning properly.
- 2.10** The length of a certain type of screws is normally distributed with expected value 4.04 cm and standard deviation 0.05 cm.
- Compute the probability, that a randomly chosen screw is **(1)** shorter than 4.00 cm, **(2)** between 3.98 cm and 4.05 cm.
  - Screws which are shorter than 3.97 cm or longer than 4.11 cm fail the quality check. Compute the percentage of rejects.
  - Another machine produces screws with expected value 4.05 cm and standard deviation 0.05 cm. It is claimed that only 5% of the produced screws exceed the tolerance limit of 4.11 cm. Is this claim true? Reason your answer!  
[normally distributed ... normalverteilt; rejects ... Ausschussware; exceed ... überschreiten; tolerance limit ... Toleranzgrenze]
- 2.11** From several quality checks it is known that 5% of a certain LED-type are dead after at most one year, and 7% function at least 3.5 years.
- Assume that the durability of the LEDs is normally distributed and compute expected value and standard deviation.
  - Compute the percentage of LEDs which have a lifespan of at least 4 years.  
[at most ... höchstens; durability/lifespan ... Lebensdauer]
- 2.12** A machine tool produces circular metal plates with a desired diameter of 15 cm. The diameter is normally distributed with standard deviation  $\sigma = 0.12$  cm. Calculate the value the machine has to be adjusted to, if ...
- ... at most 5% of the produced plates have a diameter less than 14.80 cm.
  - ... at most 2% of the produced plates have a larger diameter than 15.15 cm.  
[machine tool ... Fertigungsroboter; diameter ... Durchmesser]

## 2 Zufallsvariable

### Englische Aufgaben

**2.13** An exam consists of 50 single choice questions, each providing two possible answers to tick off. Of these two provided answers, exactly one is correct. Compute the following probabilities and argument, whether an approximation of the binomial distribution is reasonable! If you approximate the binomial distribution, use continuity correction.

**a.**  $P(\text{exactly 25 questions are answered correctly})$

**b.** For passing the test, at least 60% of the questions have to be answered correctly. Compute the probability for passing the test.

**c.** If a student answers at least 26 questions but at most 29 questions correct, he or she is allowed to take an additional oral exam. Compute the probability of being permitted to take the oral exam.

[reasonable ... sinnvoll; oral examination ... mündliche Prüfung; continuity correction ... Stetigkeitskorrektur]

**2.14** It is known from experience that 4% of the students who registered for an exam do not show at the actual test. A certain test takes place in an auditorium with 280 seats.

**a.** Calculate the number of applications which are maximally allowed, so that with a probability of 98% all actual appearing students can participate in the exam.

**b.** 295 students are registered for another test. Compute the probability that more than 280 students show at the test.

In both cases, you do not have to take continuity correction into account!

[to register ... anmelden]