1 Differentialrechnung

Englische Aufgaben

1.1 Given is a polynomial function f.

(1) Compute the derivative of f and the linear approximation h of f at the point a by writing $f(x) = f\big((x-a)+a\big)$ in the form $f(a) + c * (x-a) + u(x) * (x-a)^2$.

(2) Plot both graphs in the same coordinate system.

(3) Compare the function values $f(b)$ and $h(b)$ for $b_1$und b_2 and compute the absolute error.

a. $f(x) = \frac{1}{8}x^3 + 3; a = 2; b_1 = 2.05, b_2 = 2.6$ **b.** $f(x) = \frac{1}{4}(x^3 - 8x); a = 1; b_1 = 1.02, b_2 = 2$

[polynomial function … Polynomfunktion; derivative … Ableitung; linear approximation … lineare Näherung; to plot … darstellen; absolute error … absoluter Fehler]

1.2 Compute the slope of the tangent t to the function f in a, as well as an equation of t. Additionally, compute a parametric representation of t. Plot f and t in the same coordinate system.

a. $(x) = \frac{x+5}{x^2+1}; a = 0$ **b.** $f(x) = 0.25x^4 - 2x^2 + 2; a = 1$

[slope … Steigung; tangent … Tangente; parametric representation … Parameterform]

1.3 A car moves with constant acceleration. The distance covered after t seconds is $s(t) = 1.9t^2 + 120$ [in meters].

a. Compute the average velocity of the car between t = 3 and t = 5 in m/s and in km/h.

b. The speed limit on the cars' route is 70km/h. After 6 seconds the car passes a speed trap. Does the driver have to worry about paying a fine? Reason your answer.

c. The speedometer displays a velocity of about 130km/h. Compute the corresponding time t.

[acceleration … Beschleunigung; average … durchschnittlich; speed trap … Radarfalle; to pay a fine … Strafe zahlen; speedometer … Tachometer]

1.4 Plot the function f: $\mathbb{R} \to \mathbb{R}$, $f(x) = \frac{1}{2}x^4 - 2x^2 + 2$ and decide whether the following statements are true or false. Verify your answers by computations.

a. The difference quotient of f in [0; 1] is positive.

b. In x = 0 the differential quotient of f is zero.

c. The slope of the tangent in x = 3.5 is negative.

d. There are exactly two points with horizontal tangents.

e. The linear approximation h of f at x = -2 is given by $h: y = -8x - 14$

f. The average slope of f in $\left[-\sqrt{2};\ \sqrt{2}\right]$ is zero.

1.5 Compute a linear approximation of f in a.

a. $f(x) = -\sin\left(\frac{x}{x+1}\right); a = 0$ **b.** $f(x) = \sqrt[3]{e^x * (x^2 - 1)}; a = 0$ **c.** $f(x) = \frac{x^5 + \ln(x^2-1)}{5}; a = 2$

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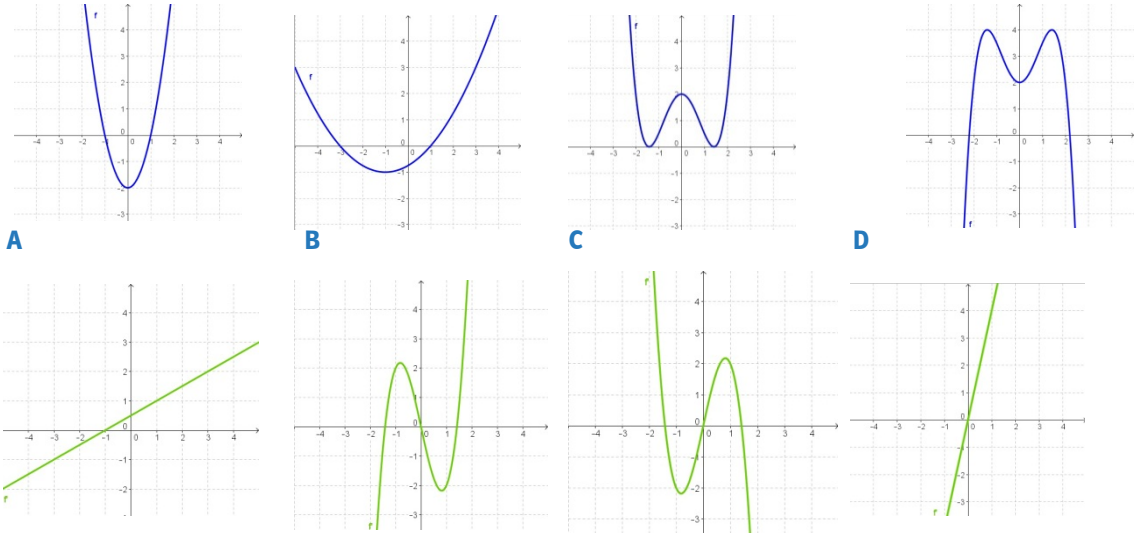
1.6 Match the function f to the corresponding graph of the derivative f'.

a. ☐ **b.** ☐ **c.** ☐ **d.** ☐



[to match … zuordnen]

1.7 Given is a differentiable function f. Solve the following tasks.

(1) Determine the largest possible domain of f and compute all zeros.

(2) Compute all local extrema of f and decide whether they are maxima or minima.

(3) Compute the intervals in which the function f is monotone increasing and decreasing, respectively.

(4) Compute the inflection points of f and the intervals in which f is convex and concave, respectively.

(5) Plot the graph of f.

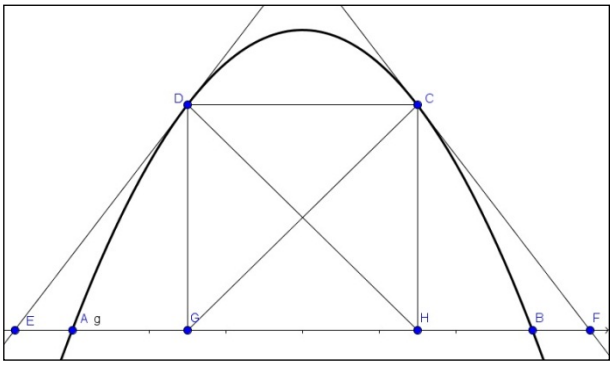
[differentiable … differenzierbar; task … Aufgabe; domain … Definitionsbereich; local extremum … lokaler Extrempunkt; inflection point … Wendepunkt]

a. $f(x) = \frac{x^2}{x^2+4}$ **b.** $f(x) = x^4 + x^3 - 6x^2$

1.8 The scaffolding of a bridge (as shown in the picture below) may be modelled by a quadratic function g(x). The distance AB is 60 m, the maximal height of the scaffolding is 15 m.

a. Find the coefficients of the function. Therefore, choose a coordinate system with origin in A.

b. For increasing stability, additional struts are used. They can be modelled using the tangents to g with slope $\pm\frac{2}{5}$. Compute the length of the horizontal strut, i.e., the distance between the intersection points C and D of the tangents and g, and the length of the diagonal struts, i.e., the distances DE and CG, respectively.



[scaffolding … Gerüst; coefficient … Koeffizient; strut … Strebe; intersection points … Schnittpunkte; i.e. (id est) … das heißt]

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1.9 The volume of a water tank shall be 500 m³. Compute the dimensions of

a. the cylindrical tank,

b. the dimensions of a prismatic tank with quadratic base

with minimal surface and compute the surface. In both cases, you do not need to consider the objects' base.

[water tank … Wassertank; cylindrical … zylindrisch; volume … Volumen; prismatic … prismatisch; surface … Oberfläche; base … Grundfläche]

1.10 Find a zero of $f(x) = \cos(3x) - x + 4$ in $[4; 7]$.

a. Perform the bisection method until the error is less than 0.0001.

b. Use Newton's method for **(1)** $x_0 = 4$, **(2)** $x_0 = 5$. Compare your results.

Compare your results from **a.** and **b.** and plot the function.

[bisection method … Bisektionsverfahren; Newton's method … Newton-Verfahren]