4 Integralrechnung

Englische Aufgaben

4.1 Which of the functions F are antiderivatives of the given polynomial, trigonometric and exponential functions f, respectively?

a. $f(x) = 3x^2$

A $F(x) = 9x^2$ **B** $F(x) = 0.75x^4$ **C** $F(x) = 3x^4 - 7$ **D** $F(x) = \frac{3}{4x^4} + 12$

b. $f(x) = \sin(3x)$

A $F(x) = -\cos\left(\frac{3}{2}x^2\right)$ **B** $F(x) = \frac{-\cos(3x^2)}{3}$ **C** $F(x) = -\frac{1}{3}\cos(3x)$ **D** $F(x) = \cos(3x)$

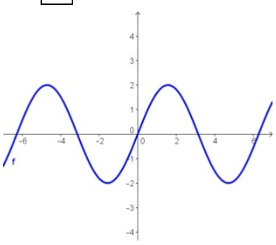
c. $f(x) = e^{3x}$

A $F(x) = e^{\frac{3x^2}{2}}$ **B** $F(x) = 3e^{3x}$ **C** $F(x) = \frac{1}{3}e^{3x}$ **D** $F(x) = \frac{e^{3x}+5}{3}$

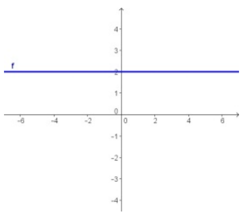
[antiderivative … Stammfunktion]

4.2 Match each function f to the graph of an appropriate primitive integral F.

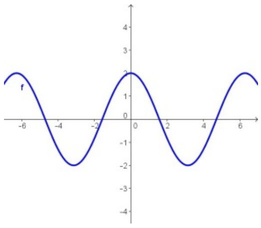
a. ☐



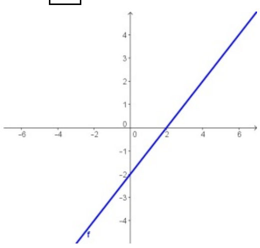
b. ☐



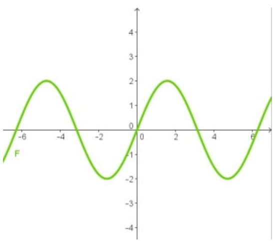
c. ☐



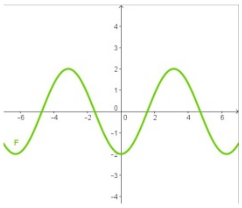
d. ☐



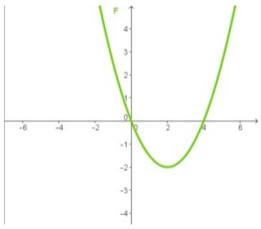
A



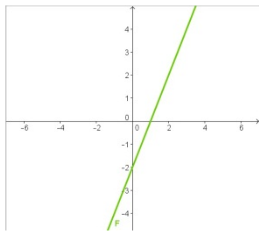
B



C



D



[primitive integral … Stammfunktion]

4.3 Draw the graph of an antiderivative F of the function f satisfying the condition $F(a) = b$.

a. $f(x) = 3x^2; a = 0, b = -2$ **b.** $f(x) = \cos x\,; a = \frac{\pi}{2}, b = 5$ **c.** $f(x) = 8x + 8; a = -4, b = 0$

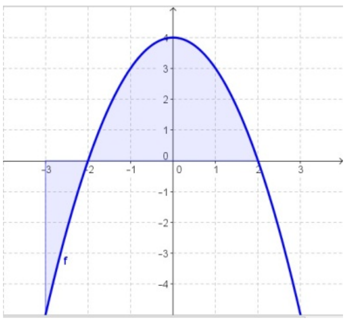
4.4 Compute the indefinite integral.

a. $\int 3x^2 - 5x + \frac{3}{x}\,dx$ **b.** $\int 3xy^2 - 5yx^3 + y\,\,dy$ **c.** $\int 5ab^5 - a + \sin(ab)\,db$

[indefinite integral … unbestimmtes Integral]

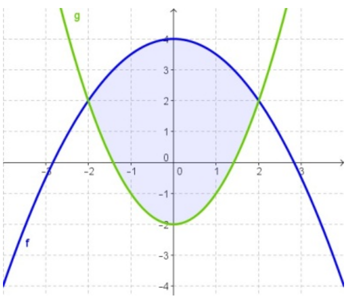
4.5 Write down a formula using definite integrals for computing the coloured area and compute the area.

a.



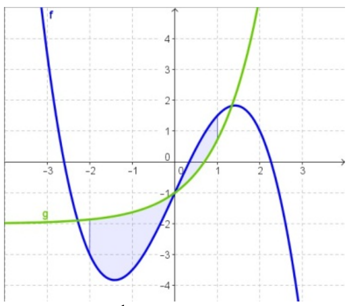
$f(x) = -x^2 + 4$

b.



$f(x) = -\frac{1}{2}x^2 + 4,\ g(x) = x^2 - 2$

c.



$f(x) = -\frac{1}{2}x^3 + 3x - 1,\ g(x) = e^x - 2$

[definite integral … bestimmtes Integral; area … Fläche]

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4 Integralrechnung

Englische Aufgaben

4.6 Plot the function $f:\mathbb{R} \to \mathbb{R}$, $f(x) = x^4 - 5x^2 + 4$. Decide whether the following statements are true or false.

a. $\int_{-2}^{1} f(x)\,dx = \int_{1}^{2} f(x)\,dx$

b. The area enclosed by f and the x-axis is $A = \int_{-2}^{2} f(x)\,dx$.

c. $\int_{-1}^{1} f(x)\,dx = 2 * \int_{0}^{1} f(x)\,dx$

d. $\int_{0}^{2} f(x)\,dx < 0$

f. The area enclosed by f and the x-axis in $[-2; 0]$ is $\left|\int_{-2}^{-1} f(x)\,dx\right| = \left|\int_{-1}^{0} f(x)\,dx\right|$.

[to enclose … einschließen; x-axis … x-Achse]

4.7 The acceleration of a car at t seconds is $a(t) = -0.0003t^2 + 0.08t + 2$for t in [0; 14].

a. Find a function v for t in [0; 14] describing the cars' velocity at time t in m/s, if the cars' velocity at t = 0 is zero, and compute the velocity at t = 7.

b. Find a function s which for all t in [0; 14] gives the covered distance in [0; t] in meters.

c. Compute the distance the car has covered after 12 sec.

d. After 14 seconds the car slows down again. The deceleration due to braking is 6.5 m/s^2. Find a function v_b which gives the velocity of the car at time t during the slowing-down process. Compute the time until the car stops as well as the covered distance.

[acceleration … Beschleunigung; covered distance … zurückgelegte Strecke; to slow down … bremsen; deceleration due to braking … Bremsverzögerung; slowing-down process … Bremsvorgang]

4.8 Compute the mean value of $f(x) = 3x^3 - 15x$ in [0; 4].

4.9 A cup of tea is left to cool down. The process of cooling down is described by the function T with $T(t) = 63e^{-0.15t} + 25$, where t is the time in minutes and T(t) the temperature of the tea in °C.

a. Compute the temperature of T at t = 0 and at t = 10.

b. How long does it take until the temperature has dropped to approximately 30°C?

c. Compute the average temperature of the tea in the first 8 minutes.

[to leave to cool down … abkühlen lassen; to drop … absinken]

4.10 We know several function values of a continuous function f: f(0) = 0; f(3) = 46.5; f(6) = 228.

a. Compute the integral approximately using **(1)** the midpoint rule, **(2)** the trapezoidal rule and **(3)** Simpson's rule.

b. Assume, the equation of f is given by $f(x) = \frac{1}{2}x^3 + 3x^2 + 2x$. Compute the exact integral of f and the absolute error between each approximation and the true value of the integral.

[midpoint rule (or rectangle rule) … Rechtecksregel; trapezoidal rule … Trapezregel; Simpson's rule[1] … Kepler'sche Fassregel; remark … Bemerkung]

4.11 Use the composite Simpson's rule and compute the integral $\int_{0}^{6} x^3 e^{-x}\,dx$ for n = 3.

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[1] **Remark:** In German, the formula $\int_a^b f(x)\,dx \approx \frac{1}{6} * (b-a) * \left[f(a) + 4 * f\left(\frac{a+b}{2}\right) + f(b)\right]$is called *Kepler'sche Fassregel* and the formula $\int_a^b f(x)\,dx \approx \frac{b-a}{6n}[f(a) + 4 * f(a_1) + 2 * f(a_2) + \cdots + 4 * f(a_{2n-1}) + f(b)]$ is called *zusammengesetzte Fassregel* or *Simpson-Regel*. However, in English the formulas are called *Simpson's rule* and *composite Simpson's rule*, respectively.

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