Lösungswege 5 – Online **Arbeitsblatt**

Thema: Trigonometrische Grundbeziehungen		**Grundkompetenz:**
Name:	**Schwierigkeitsgrad:** schwer	**Klasse:**

Zeige unter Anwendung der trigonometrischen Grundbeziehungen $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$ und $\sin^2(\alpha) + \cos^2(\alpha) = 1$ die Gültigkeit der folgenden Beziehungen zwischen den Winkelfunktionen.

a) $\cos^2(\alpha) = \frac{1}{1+\tan^2(\alpha)}$

b) $\tan^2(\alpha) = \frac{1-\cos^2(\alpha)}{\cos^2(\alpha)}$

c) $\sin(\alpha) = \sqrt{1-\cos^2(\alpha)}$

d) $\sin^2(\alpha) = \frac{1}{1+\left(\frac{\cos(\alpha)}{\sin(\alpha)}\right)^2}$

e) $\sin^2(\alpha) = \frac{\tan^2(\alpha)}{1+\tan^2(\alpha)}$

f) $\frac{1}{\sin^2(\alpha)} = 1 + \left(\frac{\cos(\alpha)}{\sin(\alpha)}\right)^2$

Thema: Trigonometrische Grundbeziehungen Lösungen		**Grundkompetenz:**
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a) $\cos^2(\alpha) = \frac{1}{1+\tan^2(\alpha)}$

$$\frac{1}{1+\tan^2(\alpha)} = \frac{1}{1+\left(\frac{\sin(\alpha)}{\cos(\alpha)}\right)^2} = \frac{1}{\frac{\cos^2(\alpha)+\sin^2(\alpha)}{\cos^2(\alpha)}} = \frac{1}{\frac{1}{\cos^2(\alpha)}} = \cos^2(\alpha)$$

b) $\tan^2(\alpha) = \frac{1-\cos^2(\alpha)}{\cos^2(\alpha)}$

$$\frac{1-\cos^2(\alpha)}{\cos^2(\alpha)} = \frac{\sin^2(\alpha)}{\cos^2(\alpha)} = \left(\frac{\sin(\alpha)}{\cos(\alpha)}\right)^2 = \tan^2(\alpha)$$

c) $\sin(\alpha) = \sqrt{1-\cos^2(\alpha)}$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1 \rightarrow \sin^2(\alpha) = 1 - \cos^2(\alpha) \rightarrow \sin(\alpha) = \sqrt{1-\cos^2(\alpha)}$$

d) $\sin^2(\alpha) = \frac{1}{1+\left(\frac{\cos(\alpha)}{\sin(\alpha)}\right)^2}$

$$\frac{1}{1+\left(\frac{\cos(\alpha)}{\sin(\alpha)}\right)^2} = \frac{1}{1+\frac{\cos^2(\alpha)}{\sin^2(\alpha)}} = \frac{1}{\frac{\sin^2(\alpha)+\cos^2(\alpha)}{\sin^2(\alpha)}} = \frac{1}{\frac{1}{\sin^2(\alpha)}} = \sin^2(\alpha)$$

e) $\sin^2(\alpha) = \frac{\tan^2(\alpha)}{1+\tan^2(\alpha)}$

$$\frac{\tan^2(\alpha)}{1+\tan^2(\alpha)} = \frac{\frac{\sin^2(\alpha)}{\cos^2(\alpha)}}{1+\frac{\sin^2(\alpha)}{\cos^2(\alpha)}} = \frac{\frac{\sin^2(\alpha)}{\cos^2(\alpha)}}{\frac{\cos^2(\alpha)+\sin^2(\alpha)}{\cos^2(\alpha)}} = \frac{\frac{\sin^2(\alpha)}{\cos^2(\alpha)}}{\frac{1}{\cos^2(\alpha)}} = \frac{\sin^2(\alpha)}{\cos^2(\alpha)} \cdot \frac{\cos^2(\alpha)}{1} = \sin^2(\alpha)$$

f) $\frac{1}{\sin^2(\alpha)} = 1 + \left(\frac{\cos(\alpha)}{\sin(\alpha)}\right)^2$

$$1 + \left(\frac{\cos(\alpha)}{\sin(\alpha)}\right)^2 = 1 + \frac{\cos^2(\alpha)}{\sin^2(\alpha)} = \frac{\sin^2(\alpha)+\cos^2(\alpha)}{\sin^2(\alpha)} = \frac{1}{\sin^2(\alpha)}$$