

# 1 Differentialrechnung

## Englische Aufgaben

**1.1** Given is a polynomial function  $f$ .

**(1)** Compute the derivative of  $f$  and the linear approximation  $h$  of  $f$  at the point  $a$  by writing  $f(x) = f((x - a) + a)$  in the form  $f(a) + c * (x - a) + u(x) * (x - a)^2$ .

**(2)** Plot both graphs in the same coordinate system.

**(3)** Compare the function values  $f(b)$  and  $h(b)$  for  $b_1$  und  $b_2$  and compute the absolute error.

$$\mathbf{a.} f(x) = \frac{1}{8}x^3 + 3; a = 2; b_1 = 2.05, b_2 = 2.6 \quad \mathbf{b.} f(x) = \frac{1}{4}(x^3 - 8x); a = 1; b_1 = 1.02, b_2 = 2$$

[polynomial function ... Polynomfunktion; derivative ... Ableitung; linear approximation ... lineare Näherung; to plot ... darstellen; absolute error ... absoluter Fehler]

**1.2** Compute the slope of the tangent  $t$  to the function  $f$  in  $a$ , as well as an equation of  $t$ . Additionally, compute a parametric representation of  $t$ . Plot  $f$  and  $t$  in the same coordinate system.

$$\mathbf{a.} (x) = \frac{x+5}{x^2+1}; a = 0 \quad \mathbf{b.} f(x) = 0.25x^4 - 2x^2 + 2; a = 1$$

[slope ... Steigung; tangent ... Tangente; parametric representation ... Parameterform]

**1.3** A car moves with constant acceleration. The distance covered after  $t$  seconds is  $s(t) = 1.9t^2 + 120$  [in meters].

**a.** Compute the average velocity of the car between  $t = 3$  and  $t = 5$  in m/s and in km/h.

**b.** The speed limit on the cars' route is 70km/h. After 6 seconds the car passes a speed trap. Does the driver have to worry about paying a fine? Reason your answer.

**c.** The speedometer displays a velocity of about 130km/h. Compute the corresponding time  $t$ .

[acceleration ... Beschleunigung; average ... durchschnittlich; speed trap ... Radarfalle; to pay a fine ... Strafe zahlen; speedometer ... Tachometer]

**1.4** Plot the function  $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \frac{1}{2}x^4 - 2x^2 + 2$  and decide whether the following statements are true or false. Verify your answers by computations.

**a.** The difference quotient of  $f$  in  $[0; 1]$  is positive.

**b.** In  $x = 0$  the differential quotient of  $f$  is zero.

**c.** The slope of the tangent in  $x = 3.5$  is negative.

**d.** There are exactly two points with horizontal tangents.

**e.** The linear approximation  $h$  of  $f$  at  $x = -2$  is given by  $h: y = -8x - 14$

**f.** The average slope of  $f$  in  $[-\sqrt{2}; \sqrt{2}]$  is zero.

**1.5** Compute a linear approximation of  $f$  in  $a$ .

$$\mathbf{a.} f(x) = -\sin\left(\frac{x}{x+1}\right); a = 0 \quad \mathbf{b.} f(x) = \sqrt[3]{e^x * (x^2 - 1)}; a = 0 \quad \mathbf{c.} f(x) = \frac{x^5 + \ln(x^2 - 1)}{5}; a = 2$$

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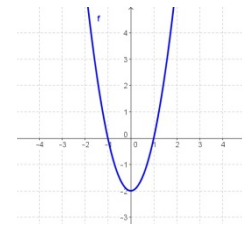
1.6 Match the function  $f$  to the corresponding graph of the derivative  $f'$ .

a.

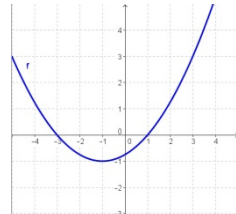
b.

c.

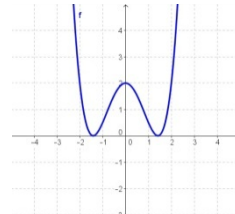
d.



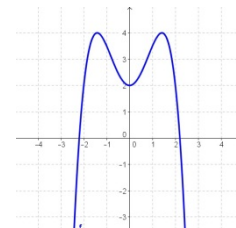
A



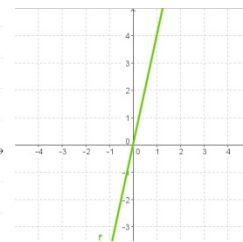
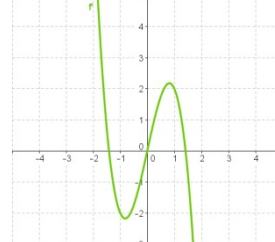
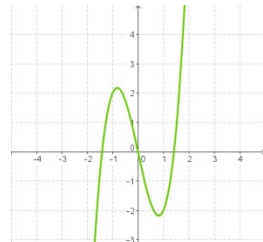
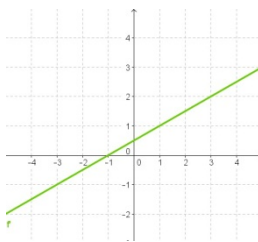
B



C



D



[to match ... zuordnen]

1.7 Given is a differentiable function  $f$ . Solve the following tasks.

- (1) Determine the largest possible domain of  $f$  and compute all zeros.
- (2) Compute all local extrema of  $f$  and decide whether they are maxima or minima.
- (3) Compute the intervals in which the function  $f$  is monotone increasing and decreasing, respectively.
- (4) Compute the inflection points of  $f$  and the intervals in which  $f$  is convex and concave, respectively.
- (5) Plot the graph of  $f$ .

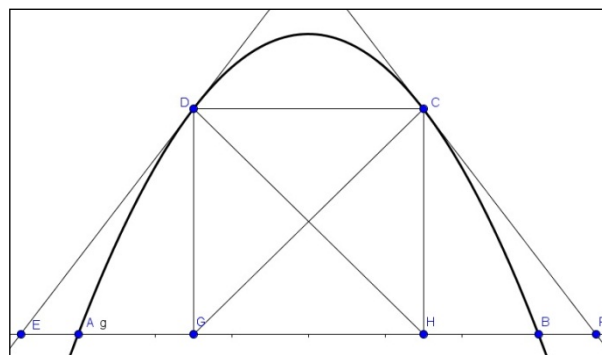
[differentiable ... differenzierbar; task ... Aufgabe; domain ... Definitionsbereich; local extremum ... lokaler Extrempunkt; inflection point ... Wendepunkt]

a.  $f(x) = \frac{x^2}{x^2+4}$

b.  $f(x) = x^4 + x^3 - 6x^2$

1.8 The scaffolding of a bridge (as shown in the picture below) may be modelled by a quadratic function  $g(x)$ . The distance AB is 60 m, the maximal height of the scaffolding is 15 m.

- a. Find the coefficients of the function. Therefore, choose a coordinate system with origin in A.
- b. For increasing stability, additional struts are used. They can be modelled using the tangents to  $g$  with slope  $\pm \frac{2}{5}$ . Compute the length of the horizontal strut, i.e., the distance between the intersection points C and D of the tangents and  $g$ , and the length of the diagonal struts, i.e., the distances DE and CG, respectively.



[scaffolding ... Gerüst; coefficient ... Koeffizient; strut ... Strebe; intersection points ... Schnittpunkte; i.e. (id est) ... das heißt]

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- 1.9** The volume of a water tank shall be  $500 \text{ m}^3$ . Compute the dimensions of
- the cylindrical tank,
  - the dimensions of a prismatic tank with quadratic base
- with minimal surface and compute the surface. In both cases, you do not need to consider the objects' base.

[water tank ... Wassertank; cylindrical ... zylindrisch; volume ... Volumen; prismatic ... prismatisch; surface ... Oberfläche; base ... Grundfläche]

- 1.10** Find a zero of  $f(x) = \cos(3x) - x + 4$  in  $[4; 7]$ .
- Perform the bisection method until the error is less than 0.0001.
  - Use Newton's method for **(1)**  $x_0 = 4$ , **(2)**  $x_0 = 5$ . Compare your results.
- Compare your results from **a.** and **b.** and plot the function.

[bisection method ... Bisektionsverfahren; Newton's method ... Newton-Verfahren]